Imaging Three-Dimensional Relative Sources from Nuclear Reactions

DAVID A. BROWN
Institute for Nuclear Theory, University of Washington, Box 351550
Seattle, WA 98195-1550, USA
E-mail: dbrown@phys.washington.edu

One can access the space-time development of a heavy-ion reaction directly by imaging the source function from two particle correlation functions. In the case of like-charged pions, this imaging can be recast as a Fourier inversion problem. We will demonstrate how this inversion can be performed on full three-dimensional (i.e. in long, side and out coordinates) experimentally determined correlation functions. We will discuss the resulting three dimensional images of the relative sources. Finally, we will discuss how to perform the full three dimensional inversion for particles whose final state interactions are more complicated than those of the pions.

1 Nuclear Interferometry

Intensity interferometry has proven to be a valuable tool for nuclear physics as it gives direct access to the space-time extent of heavy-ion reactions. The correlation function, measured in interferometry, typically is used either to extract the correlation radii or is compared to correlation functions generated from a semi-classical transport model. Recently, it was noticed that one can make better use of the correlation function – one can use it to *image* the relative source function of the particles in question [1,2]. This source function is the relative distribution of emission points of a pair of particles in their center of mass (CM) frame. The correlation radii normally extracted from the correlation functions are the widths of this distribution. While the extracted images represent an advance in the amount of information one can gather from correlation measurements, imaging was limited to angle averaged (q_{inv}) correlations. We now demonstrate the imaging of full three dimensional (3D) correlation functions.

In this talk, we will discuss how interferometry is really an inversion problem and discuss two ways to solve the full 3D problem. Both of these methods will be applied to a simulated Coulomb corrected pion correlation function. The first method is a direct application of the Fast Fourier Transform (FFT) algorithm to the correlation data. We will discuss the various problems and limitations of this algorithm. The chief limitation of this approach is that it only works for pion pairs. The second method is a 3D implementation of the general imaging procedure used in [1,2]. Unlike the FFT method, this method

works for all particle pairs. Following the comparison of these two methods, we will outline future directions for the study of 3D correlation data.

2 The Problem

Our task is to extract as much information about the source function from the two particle correlation function as we can. The experimently measured correlation function is defined as the following ratio:

$$\mathcal{R}_{\vec{P}}(\vec{q}) = C_{\vec{P}}(\vec{q}) - 1 = \frac{dN_2/d^2\vec{p_1}\vec{p_2}}{dN_1/d\vec{p_1}dN_1/d\vec{p_2}} - 1. \quad (1)$$

As stated above, the source function is the relative distribution of emission points in the pair CM and in the Pratt-Koonin formalism it is [3,4]

$$S_{\vec{P}}(\vec{r}) \equiv \int dt_1\, dt_2 \int d^3R\ \sigma\left(\vec{R} + \vec{r}/2, t_1, \vec{P} = 0\right) \sigma\left(\vec{R} - \vec{r}/2, t_2, \vec{P} = 0\right). \quad (2)$$

Here, $\sigma(\vec{r}, t, \vec{p})\, d^3r\, dt$ is the probability for emitting one of the particles at time t at position $\vec{r}$ with momentum $\vec{p}$. One should note that all time dependence is integrated out of the source function. For the purpose of this talk, the details of σ are not important.

We can extract the source function directly from the data because the source function and the correlation function are related through the Pratt-Koonin Equation [3,4]:

$$\mathcal{R}_{\vec{P}}(\vec{q}) = \int d^3r\ \left(\left|\Phi^{(-)}_{\vec{q}}(\vec{r})\right|^2 - 1\right) S_{\vec{P}}(\vec{r}) \equiv \int d^3r\ \ K(\vec{q}, \vec{r}) S_{\vec{P}}(\vec{r}). \quad (3)$$

Here $\Phi^{(-)}_{\vec{q}}(\vec{r})$ is the pair (anti-)symmetrized wave function in the pair CM frame. One should note that this is a simple integral equation with a kernel $K(\vec{q}, \vec{r})$. In the next few sections we will demonstrate the different ways to invert this integral equation.

3 Extracting the Source with the FFT

In the case of pions, the task of imaging is simple. Here, we can ignore final state interactions in the relative wavefunction, turning the problem into a Fourier transform problem. When one does this, the kernel $K(\vec{q}, \vec{r})$ becomes $K(\vec{q}, \vec{r}) = \cos(2\vec{q} \cdot \vec{r})$ and we can immediately solve for the source:

$$S_{\vec{P}}(\vec{r}) = \frac{1}{\pi^3} \int d^3q\ \cos(2\vec{q} \cdot \vec{r}) \mathcal{R}_{\vec{P}}(\vec{q}). \quad (4)$$

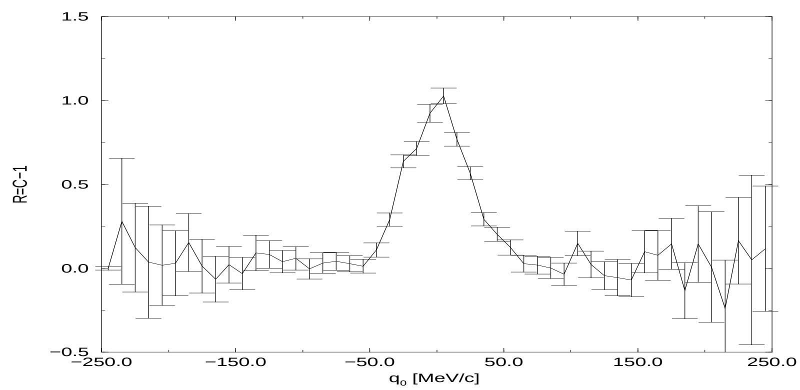

Figure 1: Sample plot of the model correlation function. This plot is along the $q_L = q_S = 0$ axis.

This integral can then be performed with the Fast Fourier Transform algorithm.

The implementation of the FFT algorithm is simple. First, we discretize eq. (4), converting this equation to a finite Fourier transform. Second, we perform the resulting transform with a canned FFT routine [5]. To compute the errors, we Monte Carlo sample the errors on the correlation function to generate a test correlation, invert the test correlation, then repeat. After test 100 runs, we compute the standard deviation of the ensemble of test sources.

Now, while the FFT is fast, there are several problems we must deal with. The first is dealing with statistical noise. Typically this is remedied by using a filter such as the Weiner optimal filter [5]. The second problem is that the data must have the number of points equal to a power of two. In other words, we need to pad data to make the next power of two, artificially increasing the resolution. The final problem is that the FFT approach only works for pion pairs.

We can now test this inversion method. To do this, we use a Gaussian test correlation with radius parameters $R_O = R_S = 4$ fm and $R_L = 6$ fm. We take the error bars from a real data set and then add statistical noise to the correlation to simulate an actual experimental (Coulomb corrected) correlation. Fig. 1 is a sample of this full correlation function. In fig. 2a. we plot the results of the inversion using the FFT alone. As one can see, the tails overestimate the true source by several orders of magnitude. What is happening here is the FFT routine is Fourier transforming the statistical noise in the data. In fig. 2b. we illustrate how the situation improves when one uses the Weiner optimal filter [5]. While the tails are now much closer to the input

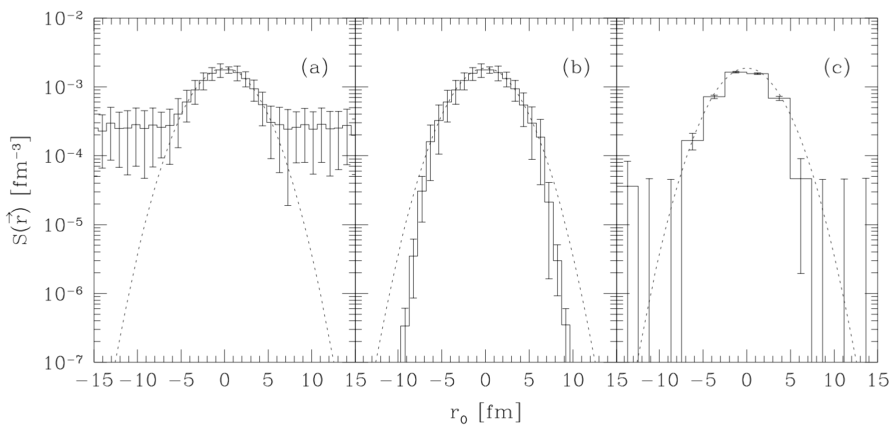

Figure 2: Sample plots of the source functions. These plots are all along the $r_L = r_S = 0$ axis. In all three panels, the dashed curve is the input source. In panel a., the source was imaged using the FFT algorithm alone. In panel b., the source was imaged using the FFT algorithm and the filter described in the appendix. In panel c., the source was imaged using the general procedure.

source, they are now well below the input source. In both cases, the inverted source *is not consistent with the input source.* A more sophisticated filtering method could likely fix this, but given the limitations of the FFT method, it is easier to abandon it entirely.

4 The General Approach

The authors of [1,2] give an approach to inverting the Pratt-Koonin equation that is applicable for any particle pair and we will now outline it. We seek the source that best represents the experimental data. Making a correlation out of a test source, $\mathcal{R}_i^{test} - 1 = \sum_j K_{ij} S_j^{test}$, we would say that the test source reproduces the data well if it is the one that minimizes the χ^2:

$$\chi^2 = \sum_j \left[\frac{(\mathcal{R}^{exp}(q_j) - \mathcal{R}^{test}(q_j))}{\Delta \mathcal{R}_j^{exp}} \right]^2 = \text{min.} \tag{5}$$

To find the minimum, we set $\delta\chi^2/\delta S_k = 0$ giving a matrix equation for the source:

$$S_j = [(K^{\mathrm{T}}[\Delta^2 \mathcal{R}^{exp}]^{-1} K)^{-1} K^{\mathrm{T}} [\Delta^2 \mathcal{R}^{exp}]^{-1} \mathcal{R}^{exp}]_j. \tag{6}$$

Similarly, by performing the error analysis one finds the covariance matrix of the source: $\Delta^2 S_{ij} = [K^T[\Delta^2 \mathcal{R}^{exp}]^{-1}K]^{-1}_{ij}$.

We apply this approach to the correlation described above. In fig. 2c., we plot the source imaged this way along with the input source. One can see that this source does as good a job at reproducing the peak of the source as the FFT sources. Also, this technique does as good at reconstructing the tails as the FFT technique. However, *unlike* the FFT, this technique returns errors that are much more realistic. In other words, while the FFT claims it can image even below noise level, the general approach admits it can not image that well.

Final Comments

We have learned several lessons by this comparison of the FFT and general approaches. While the FFT is fast it does not reliably report the uncertainty in the imaging. On the other hand, the general method does reliably estimate the uncertainty. Finally, while the FFT approach only works for pions, the general approach works for *any pair*.

There are many questions yet to answer using the 3D sources. Do pions have a non-Gaussian tails due to resonances? Proton one dimensional sources are not Gaussian, so what do they look like in 3D? How does flow effect all 3D correlations? Given that the general method works for any pair, how about 3D unlike pair correlations?

As a final note, we invite readers to download and test our one dimensional inversion code. It is available at:
http://www.phys.washington.edu/∼dbrown/HBTprogs.html

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